Observations on CCSSM Standards for Mathematical Content: What Content Is Visibly Emphasized?

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A new content model for mathematics

In the decades prior to CCSSM, mathematics standards in the United States were built on the strand model.[1] In the strand model, content strands stretched uniformly over grades preK–12. See Table 1. With the advent of CCSSM, the strand model has been modified. See Table 2.

CCSSM's reorganization of the content of school mathematics is prominently advertised by the domain names it uses. Yet this feature of the standards is also easily overlooked, if we hurry past it to examine the details of a single grade or standard. As important as it is to get the details right, the big picture matters too—and the standards' new content model is worth our attention for the way in which it reveals what content is emphasized in the standards.

Table 1. The Pre-CCSSM Strand Picture of Mathematical Content

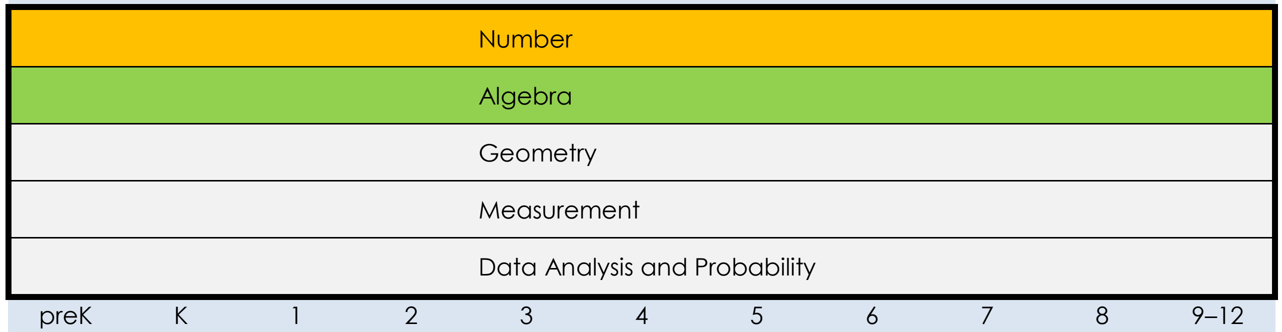
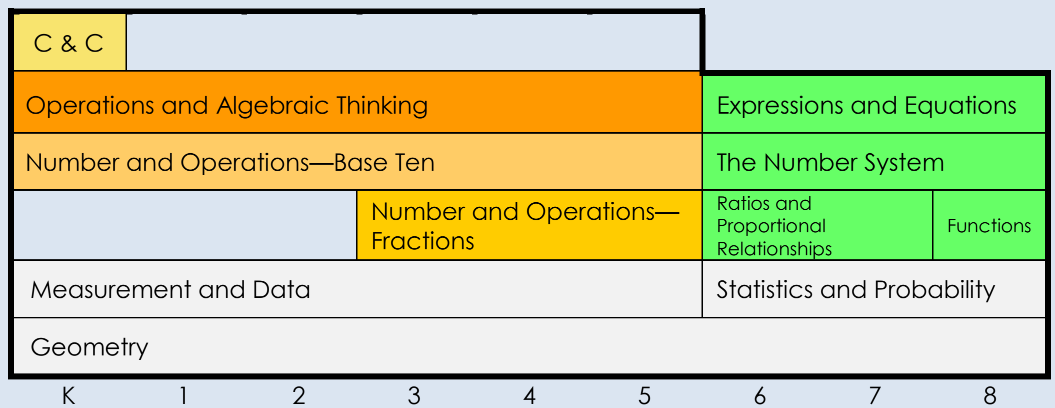


Table 2. The CCSSM Domain Picture of Mathematical Content



[1] Liping Ma (2013), "A Critique of the Structure of U.S. Elementary School Mathematics," *Notices of the American Mathematical Society*, Vol. 60, No. 10, pp. 1282–1296

Ways to describe the emphases in the content standards (1): Domain-level

Table 3. Domain-Level Emphases

K	CC, OA, NBT
1	OA, NBT
2	OA, NBT
3	OA, NBT, NF
4	OA, NBT, NF
5	OA, NBT, NF
6	RP, NS, EE
7	RP, NS, EE
8	NS, EE, F

The emphases of the standards can be read directly from the pattern of which historical content strands were allotted multiple domains in CCSSM, and which were not. This exercise reveals the content domains where CCSSM has "zoomed in" compared to the previous model—areas, that is, where the content outline itself directs top-level attention to less material (literally signaling depth over breadth). Specifically, a comparison of Tables 1 and 2 produces the emphases shown in Table 3.[2]

These particular domains are also identifiable by another method: they are the top-level organizers that are more specific than the top-level organizers in previous state standards. For example, I cannot recall any previous set of state standards that used Fractions as a top-level organizer; instead, Fractions in the past were lumped with other topics under one heading, such as Number and Operations.

Table 3 can be tested in yet another way, since lists reliably signal their own emphases by putting first things first. The four or five content domains at each grade in CCSSM are presented in the form of a list; thus, at each grade level, imagine recording the first three domains you see (the first two domains in grades 1 and 2, since these grades have fewer domains than the others). What will be the result of this exercise? Precisely Table 3.

A simple summary of the domains in Table 3 would read as follows: *the emphasis in grades K–5 is on number and operations. The emphasis in grades 6–8 is on pre-algebra, algebra, and functions*. This summary reveals the central arc of the Common Core, which points from arithmetic to algebra.

The reason the standards were designed this way has to do with their purpose. Governors and superintendents who joined together to create CCSS wanted standards that could serve new purposes compared to previous standards—principally, fostering college and career readiness and setting a globally competitive standard. Qualifying for credit-bearing courses requires thorough knowledge of algebra, and in high performing countries, large fractions of students learn a great deal of algebra. Moreover, command of numbers and early algebra is disproportionately important for careers: the first recommendation of an NCEE study of work readiness was that "A very high priority should be given to the improvement of the teaching of proportional relationships including percent, graphical representations, functions, and expressions and equations in our schools, including their application to concrete practical problems."[3] (As far as grades K–8 are concerned, everything in this recommended list of priorities belongs to Table 3.) It is for all these reasons that the standards emphasize arithmetic, algebra, and the connections between them. The standards' visible focus derives from their stated purpose.

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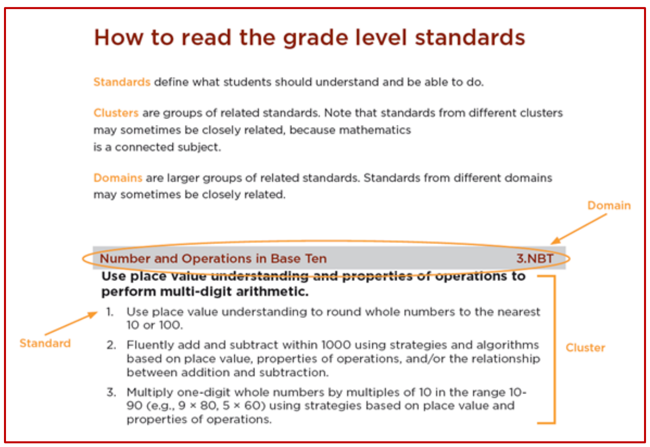
[2] In grades K–5, Number was split into four different domains (CC, OA, NBT, NF), but Measurement (MD) was not split and Geometry (G) was not split. (One can either leave K–5 Algebra aside in the analysis, or identify it with OA, as one prefers.) In grades 6–8, Algebra was split (EE, F, some of RP), as was Number (NS, some of RP), but Geometry (G) and Statistics and Probability (SP) were not split.

[3] Phil Daro, Solomon Garfunkel, *et al.*, "What Does It Really Mean to Be College and Work Ready? The Mathematics Required of First Year Community College Students." National Council on Education and the Economy, 2013.

Ways to describe the emphases in the content standards (2): Cluster-level

It is not always sufficient to work at the domain level. For example, mathematics educators will notice that area models showing the distributive property of multiplication over addition are a key part of the arc from arithmetic to algebra, yet they are missing from Table 3 (since area models of multiplication first appear in the content standards under the MD domain). To ensure that the curriculum enjoys the coherence of the standards themselves, one should ideally approach the question of emphases at the cluster level in the content standards. (CCSSM defines "domain" and "cluster" in the manner shown in this screenshot.)



Thus, one way to describe the content emphases at a finer "grain size" than the domain level is to trace the arc from arithmetic to algebra at the cluster level. For the benefit of implementation, the clusters that participate directly in the progression from arithmetic to algebra have been listed in Table 1 of the K–8 Publishers' Criteria, a document developed by the CCSSM writing team with review and collaboration from partner organizations, individual experts, and districts. This information is available at www.corestandards.org, and it is shown below in a screenshot.

Table 1. Progress to Algebra in Grades K–8

K	1	2	3	4	5	6	7	8
Know number names and the count sequence Count to tell the number of objects Compare numbers Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from Work with numbers 11-19 to gain foundations for place value	Represent and solve problems involving addition and subtraction Understand and apply properties of operations and the relationship between addition and subtraction Add and subtract within 20 Work with addition and subtraction equations Extend the counting sequence Understand place value Use place value understanding and properties of operations to add and subtract Measure lengths indirectly and by iterating length units	Represent and solve problems involving addition and subtraction Add and subtract within 20 Understand place value Use place value understanding and properties of operations to add and subtract Measure and estimate lengths in standard units Relate addition and subtraction to length	Represent & solve problems involving multiplication and division Understand properties of multiplication and the relationship between multiplication and division Multiply & divide within 100 Solve problems involving the four operations, and identify & explain patterns in arithmetic Develop understanding of fractions as numbers Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects Geometric measurement: understand concepts of area and relate area to multiplication and to addition	Use the four operations with whole numbers to solve problems Generalize place value understanding for multi-digit whole numbers Use place value understanding and properties of operations to perform multi-digit arithmetic Extend understanding of fraction equivalence and ordering Build fractions from unit fractions by applying and extending previous understandings of operations Understand decimal notation for fractions, and compare decimal fractions	Understand the place value system Perform operations with multi-digit whole numbers and decimals to hundredths Use equivalent fractions as a strategy to add and subtract fractions Apply and extend previous understandings of multiplication and division to multiply and divide fractions Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition Graph points in the coordinate plane to solve real-world and mathematical problems*	Apply and extend previous understandings of multiplication and division to divide fractions by fractions Apply and extend previous understandings of numbers to the system of rational numbers Understand ratio concepts and use ratio reasoning to solve problems Apply and extend previous understandings of arithmetic to algebraic expressions Reason about and solve one-variable equations and inequalities Represent and analyze quantitative relationships between dependent and independent variables	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers Analyze proportional relationship and use them to solve real-world and mathematical problems Use properties of operations to generate equivalent expressions Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Work with radical and integer exponents Understand the connections between proportional relationships, lines, and linear equations** Analyze and solve linear equations and pairs of simultaneous linear equations Define, evaluate, and compare functions Use functions to model relationships between quantities

*Indicates a cluster that is well thought of as part of a student's progress to algebra, but that is currently not designated as Major by one or both of the assessment consortia in their draft materials. Apart from the asterisked exception, the clusters listed here are a subset of those designated as Major in both of the assessment consortia's draft documents. ** Depends on similarity ideas from geometry to show that slope can be defined and then used to show that a linear equation has a graph which is a straight line and conversely.

Ways to describe the emphases in the content standards (3): Complete picture

Not all of the clusters in the content standards appear in the Progress to Algebra table. What is one to make of the other clusters? Some clues can be found in the specific language of certain standards from domains that fall outside of the Progress to Algebra continuum. Consider the following Kindergarten standard:

Measurement and Data | **K.MD**

Classify objects and count the number of objects in each category.

3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[3]

This standard is about data, of course, but notice the refrain of *counting*, which builds in a connection to the Counting and Cardinality domain. Standard K.MD.3, when read carefully, prompts us to use data work in *support* of counting and cardinality.

More generally, the content clusters in grades K–8 can be usefully sorted, in a focused and coherent way, into three kinds: Major, Additional, and Supporting. Major clusters coherently capture the emphases at each grade; Supporting clusters have natural connections to Major clusters; and Additional clusters stand well on their own, with weaker natural connections to the Major clusters. These designations are attached here as an appendix.

Making good use of the Critical Areas texts

The Critical Areas texts in CCSSM complement the Standards for Mathematical Content in several ways. Thanks to their narrative form, the Critical Areas texts provide richer detail about sense-making than is possible to convey in individual standards statements. These texts also describe relationships between standards that belong to different clusters (see grade 4, for example). And they make cross-grade connections explicit (note, for example, the way the grade 2 Critical Areas text foreshadows later-grades topics such as area).[4] *Coherence*, then, is a strength of the Critical Areas texts, both within and across grades.

However, the same narrative format that makes the Critical Areas texts a good vehicle for describing examples of coherence has drawbacks when it comes to systematically describing the emphases in the content standards. For example, the grade 2 Critical Areas text omits addition and subtraction word problems, even though grade 2 is when students will be facing the most difficult types of addition and subtraction word problems (see the *Progression* document for the OA domain.) The grade 7 Critical Areas text mentions equations in its introductory note, but then equations disappear in the longer discussion about the grade. In grade 5, the Critical Areas text makes fluency the goal when adding unlike denominators, whereas domain 5.NF itself does not. These and other matters are handled systematically when we work with Major, Additional, and Supporting clusters.

None of this is intended as a criticism of the Critical Areas texts, by the way; there's only so much that a brief page of prose can accomplish. And at the end of the day it is good to know the Critical Areas texts agree qualitatively with the focus on major work, as three-quarters of the verbiage in the Critical Areas texts applies to Major clusters.

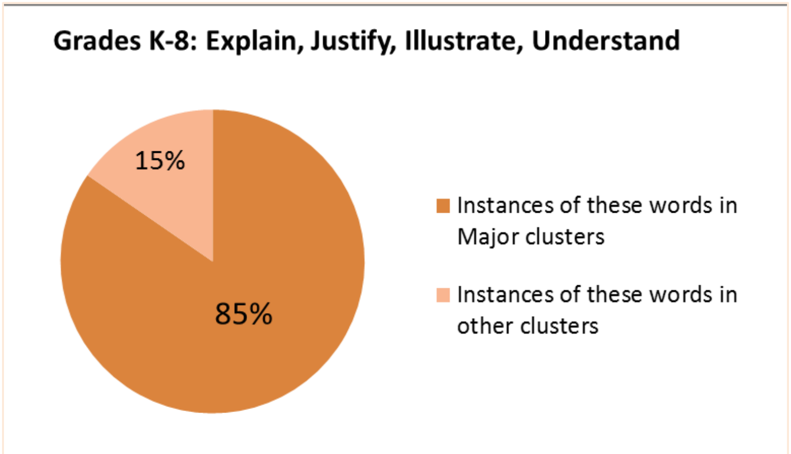
Focusing on major work strengthens the Standards for Mathematical Practice

Many educators feel that what is most special about CCSSM is the Standards for Mathematical Practice. This view too is consistent with focusing on major work, and indeed it requires focusing on major work, because practice standards imply nothing if not depth in the approach to content.

Nothing about the content standards' emphasis on major work threatens the practice standards. Indeed, the opposite is

[4] More information about such multi-grade coherences can be found in the *Progressions* documents.

true: focusing on major work aids the implementation of the practice standards because of the way the standards in Major clusters disproportionately require depth. Here is a graph showing that words like *explain* and *justify* are overwhelmingly concentrated in Major clusters relative to Additional or Supporting clusters:



During the writing of the standards, pains were evidently taken to include words like *understand* preferentially in major work, as opposed to other work, in the same way pains were taken to include words like *count* in a data standard and not the reverse.

By a simple count, the Major clusters are over half of the total (58%). But a simple count of the Major clusters is misleading because as the graph shows, mere counting fails to reveal the visibly deeper treatment the standards give to this material.

Implementing the practice standards is consistent with focusing on major work, and conversely, not focusing on major work slights the specific ways in which the content standards emphasize mathematical practices.

Summary and conclusion

Here are three systematic ways to describe the emphases in the content standards, based on the standards' visible features, their mathematical structure, and their stated design purpose:

- The emphasized domains shown in Table 3
- The Progress to Algebra clusters shown in the K–8 Publishers' Criteria on www.corestandards.org
- The Major, Additional, and Supporting clusters listed in the appendix of this document.

All three of these ways are valuable to ensure that implementation of the standards focuses where the standards do. The three approaches offer increasing levels of completeness and precision, with the most complete and precise version being the Major, Additional, and Supporting clusters.

Appendix: Cluster-level emphases and structure of the Standards for Mathematical Content

Not all content in a given grade is emphasized equally in the Standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. More time in these areas is also necessary for students to meet the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the Standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.

The content clusters in grades K–8 can be usefully sorted, in a focused and coherent way, into three kinds: Major (M), Additional (A), and Supporting (S). Major clusters capture the emphases at each grade; Supporting clusters have natural connections to Major clusters; and Additional clusters stand well on their own, with weaker natural connections to the Major clusters.

K.CC.A	M	Know number names and the count sequence.
K.CC.B	M	Count to tell the number of objects.
K.CC.C	M	Compare numbers.
K.OA.A	M	Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.
K.NBT.A	M	Work with numbers 11–19 to gain foundations for place value.
K.MD.A	A	Describe and compare measureable attributes.
K.MD.B	S	Classify objects and count the number of objects in categories.
K.G.A	A	Identify and describe shapes.
K.G.B	S	Analyze, compare, create, and compose shapes.

1.OA.A	M	Represent and solve problems involving addition and subtraction.
1.OA.B	M	Understand and apply properties of operations and the relationship between addition and subtraction.
1.OA.C	M	Add and subtract within 20.
1.OA.D	M	Work with addition and subtraction equations.
1.NBT.A	M	Extending the counting sequence.
1.NBT.B	M	Understand place value.
1.NBT.C	M	Use place value understanding and properties of operations to add and subtract.
1.MD.A	M	Measure lengths indirectly and by iterating length units.
1.MD.B	A	Tell and write time.
1.MD.B	S	Represent and interpret data.
1.G.A	A	Reason with shapes and their attributes.

2.OA.A	M	Represent and solve problems involving addition and subtraction.
2.OA.B	M	Add and subtract within 20.
2.OA.C	S	Work with equal groups of objects to gain foundations for multiplication.
2.NBT.A	M	Understand place value.
2.NBT.B	M	Use place value understanding and properties of operations to add and subtract.
2.MD.A	M	Measure and estimate lengths in standard units.
2.MD.B	M	Relate addition and subtraction to length.
2.MD.C	S	Work with time and money.
2.MD.D	S	Represent and interpret data.
2.G.A	A	Reason with shapes and their attributes.

3.OA.A	M	Represent and solve problems involving multiplication and division.
3.OA.B	M	Understand properties of multiplication and the relationship between multiplication and division.
3.OA.C	M	Multiply and divide within 100.
3.OA.D	M	Solve problems involving the four operations, and identify and explain patterns in arithmetic.
3.NBT.A	A	Use place value understanding and properties of operations to perform multi-digit arithmetic.
3.NF.A	M	Develop understanding of fractions as numbers.
3.MD.A	M	Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
3.MD.B	S	Represent and interpret data.
3.MD.C	M	Geometric measurement: understand concepts of area and

		relate area to multiplication and to addition.
3.MD.D	A	Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.
3.G.A	S	Reason with shapes and their attributes.
4.OA.A	M	Use the four operations with whole numbers to solve problems.
4.OA.B	S	Gain familiarity with factors and multiples.
4.OA.C	A	Generate and analyze patterns.
4.NBT.A	M	Generalize place value understanding for multi-digit whole numbers.
4.NBT.B	M	Use place value understanding and properties of operations to perform multi-digit arithmetic.
4.NF.A	M	Extend understanding of fraction equivalence and ordering.
4.NF.B	M	Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.C	M	Understand decimal notation for fractions, and compare decimal fractions.
4.MD.A	S	Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
4.MD.B	S	Represent and interpret data.
4.MD.C	A	Geometric measurement: understand concepts of angle and measure angles.
4.G.A	A	Draw and identify lines and angles, and classify shapes by properties of their lines and angles.
5.OA.A	A	Write and interpret numerical expressions.
5.OA.B	A	Analyze patterns and relationships.
5.NBT.A	M	Understand the place value system.
5.NBT.B	M	Perform operations with multi-digit whole numbers and with decimals to hundredths.
5.NF.A	M	Use equivalent fractions as a strategy to add and subtract fractions.
5.NF.B	M	Apply and extend previous understandings of multiplication and division to multiply and divide fractions.
5.MD.A	S	Convert like measurement units within a given measurement system.
5.MD.B	S	Represent and interpret data.
5.MD.C	M	Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.
5.G.A	A	Graph points on the coordinate plane to solve real-world and mathematical problems.
5.G.B	A	Classify two-dimensional figures into categories based on their properties.
6.RP.A	M	Understand ratio concepts and use ratio reasoning to solve problems.
6.NS.A	M	Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
6.NS.B	A	Compute fluently with multi-digit numbers and find common factors and multiples.
6.NS.C	M	Apply and extend previous understandings of numbers to the system of rational numbers.
6.EE.A	M	Apply and extend previous understandings of arithmetic to algebraic expressions.
6.EE.B	M	Reason about and solve one-variable equations and inequalities.
6.EE.C	M	Represent and analyze quantitative relationships between dependent and independent variables.
6.G.A	S	Solve real-world and mathematical problems involving area, surface area, and volume.
6.SP.A	A	Develop understanding of statistical variability.
6.SP.B	A	Summarize and describe distributions.
7.RP.A	M	Analyze proportional relationships and use them to solve real-world and mathematical problems.
7.NS.A	M	Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
7.EE.A	M	Use properties of operations to generate equivalent expressions.
7.EE.B	M	Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
7.G.A	A	Draw, construct and describe geometrical figures and describe the relationships between them.

7.G.B	A	Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.
7.SP.A	S	Use random sampling to draw inferences about a population.
7.SP.B	A	Draw informal comparative inferences about two populations.
7.SP.C	S	Investigate chance processes and develop, use, and evaluate probability models.
8.NS.A	S	Know that there are numbers that are not rational, and approximate them by rational numbers.
8.EE.A	M	Work with radicals and integer exponents.
8.EE.B	M	Understand the connections between proportional relationships, lines, and linear equations.
8.EE.C	M	Analyze and solve linear equations and pairs of simultaneous linear equations.
8.F.A	M	Define, evaluate, and compare functions.
8.F.B	M	Use functions to model relationships between quantities.
8.G.A	M	Understand congruence and similarity using physical models, transparencies, or geometry software.
8.G.B	M	Understand and apply the Pythagorean Theorem.
8.G.C	A	Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.
8.SP.A	S	Investigate patterns of association in bivariate data.